

## Hardy-Weinberg Equilibrium Equation Problems

The Hardy-Weinberg equation is a useful tool to use to determine if evolution has occurred. Any changes in gene frequency over time can be detected. Usually there is evolution occurring since all five of the Hardy-Weinberg equation conditions are rarely ever met. There are a lot of tricky ways for me to give you Hardy-Weinberg problems that might make it seem like you do not understand the concept because of deficiencies in your math training; I will not do that. On the other hand, I do want you to be able to use the formula and detect evolutionary change in at least two example ways. The first only gives you the total population and hints or outright gives you the number of homozygous individuals. The second gives you the total population and the number of individuals with homozygous dominant, heterozygous and homozygous alleles. The third may appear most commonly on future tests; it gives you one allele's value and asks you about some other value. For example, it gives you a value for a and asks for Aa. **YOU MUST BE ABLE TO DO THESE THREE TYPES FOR THE TEST.** As you will some day see, there are more ways to ask Hardy-Weinberg questions, many more. Below are examples, worked out completely and practice problems of the three types.

***DO the practice problems BEFORE the test.***

### Type One – Total Population and Number of Homozygous Recessive Individuals

Example: Assume a hypothetical population of plants meets all criteria of the Hardy-Weinberg equilibrium. Fruit color is controlled by a single locus (trait or gene) which shows complete dominance; blue fruit morph is dominant over white fruit morph. In a population of 200 plants, you count 18 white-fruited plants and 182 blue fruited plants. How many blue fruited plants are genotype Aa and how many are genotype AA.

Hint: you know  $q^2$  in the Hardy-Weinberg Equation!!!  $q^2$  is 18/200 or 0.09 (aa = white and there are 18 white-fruited plants).

#### Steps: (practice these on your calculator as you read)

1. Divide number of homozygous recessive individuals by total population:  $18/200 = 0.09$
2. You know  $q^2$  from question 1 ( $q^2 = 0.09$ )
3. Find q by finding square root of  $q^2$   $\sqrt{0.09} = 0.3$  (use a calculator)
4. Find p from basic Hardy-Weinberg equation ( $p+q=1$ ) so  $p = 1-q$ .  $P = 1-0.3$  or 0.7
5.  $p^2$  = number of homozygous dominant individuals (AA).  $p^2 = (0.7)^2 = 0.49$
6. To find out how many of the 200 plants are AA multiply  $0.49 \times 200 = 98$
7.  $2pq$  = number of heterozygous individuals (Aa).  $2pq = 2*(0.7)*(0.3) = 0.42$
8. To find out how many of the 200 plants are Aa multiply  $0.42 \times 200 = 84$
9. Check if the separate genotypes in your work add up to the total population:  $98 + 84 + 18 = 200$  (hint: if it does not math may be a problem for you; work it out again until it does work)

#### Type I - Practice problems

	Total population	Homozygous recessive individuals	Genes involved
1.	300	64	TT, Tt and tt
2.	1000	250	BB, Bb and bb
3.	27	4	RR, Rr and rr

- Answers:
1. TT = 87, Tt=149, tt is of course 64  
( $q^2 = 0.21333$ ,  $q = 0.46188$ ,  $p = 0.53812$ ,  $p^2 = 0.28957$ ,  $2pq = 0.49709$ )
  2. BB = 250, Bb = 500, and bb = 250
  3. RR = 10, Rr = 13 and rr = 4

## Type Two – They give you the total population and number of each genotype in the population

If there is a population of 6129 individuals, and of those: 1787 are MM, 3037 are Mm and 1305 are mm compute the following:

The number of M alleles, p, number of m alleles, q, check that  $p+q=1$  in your work, calculate  $p^2$ ,  $2pq$ , and  $q^2$ . Also calculate the Hardy-Weinberg equilibrium frequencies and compare these with the sample. Indicate if you think the population is evolving!

This sounds kind of hairy and is until you do a couple of practice problems; have faith! Try each calculation yourself using a calculator in the following example.

### Steps: (practice these on your calculator as you read)

1. Number of M alleles is equal to  $((MM \times 2) + Mm)$  or  $(1787 \times 2) + 3037 = 6611$
2.  $p = M \text{ alleles} / (\text{total population} \times 2)$  or  $6611 / (6129 \times 2) = 0.539$
3. Number of m alleles is equal to  $((mm \times 2) + Mm)$  or  $(1305 \times 2) + 3037 = 5647$
4.  $q = m \text{ alleles} / (\text{total population} \times 2)$  or  $5647 / (6129 \times 2) = 0.461$
5. **Check: answer from 2 + 4 should equal 1**
6. Frequency of MM =  $p^2 = (0.539)^2 = 0.291$
7. Frequency of Mm =  $2pq = 2 \times 0.539 \times 0.461 = 0.497$
8. Frequency of mm =  $q^2 = (0.461)^2 = 0.212$
9. **Actual Hardy-Weinberg Frequencies calculations:**
10. Frequency of MM x total population or  $0.291 \times 6129 = 1784$
11. Frequency of Mm x total population or  $0.497 \times 6129 = 3046$
12. Frequency of mm x total population or  $0.212 \times 6129 = 1299$
13. **Compare values (10-12) with values given in the problem if different, population is evolving:**
14. MM = 1787 vs. 1784; Mm = 3037 vs. 3046; 1305 vs. 1299. (your values may be slightly off if rounded in any step)
15. The values in #14 are very close so the population is probably NOT evolving.

Wow that was a lot! Try a couple more and get better each time.

### Type II - Practice problems

Total Population	Homozygous Dominant	Heterozygous	Homozygous Recessive
1. 1000	TT = 490	Tt=420	tt=90
2. 680	BB=192	Bb=410	bb=78
3. 215	RR=106	Rr=44	rr=65

**Type Three – They give either a or A and ask you to compute homozygous dominant, heterozygotes or homozygous recessive frequencies.**

I saved this one to last because I wanted something easy for you. This may be one of the most common types of Hardy-Weinberg problems seen on tests. In each problem you will be given either (a or q) or (A or p). You will be required to determine the other members of the formula  $1 = p^2 + 2pq + q^2$ . Don't get scared, it is really not that hard. Understand and REMEMBER the following truths:

$$1 = p^2 + 2pq + q^2$$

$$1 = p + q$$

$$a = q, \text{ and } A = p$$

$$\text{Homozygous dominant} = p^2$$

$$\text{Heterozygotes} = 2pq$$

$$\text{Homozygous recessive} = q^2$$

The basis of understanding these is best shown by an example.

Question – In a population of 119 dogfish, the frequency of a recessive gene (a) is 0.25 in a particular population. Please compute the frequency of heterozygotes in the population.

How to solve it: From the problem you know that you have a value for q because  $a=q$ . The first step is to determine p. Algebra converts  $1 = p + q$  to  $p = 1.0 - q$  or  $p = 1.0 - 0.25$  or 0.75. The question asks you to determine the heterozygotes so that is  $2pq$ . You know p and q and can multiply by 2 so  $2pq = 2(0.25)(0.75) = 0.375$ .

You must slow down with these problems and not get tricked by all of the letters in each question.

Second example – In a population in Hardy-Weinberg equilibrium, there are two alleles, A and a, for a particular gene. If the frequency of the a allele is 0.39, what is the frequency of genotype AA?

How to solve it: From the problem you know that  $q = 0.39$  because  $q=a$  in all problems. To find p use this formula:  $p + q = 1$ , but use algebra to turn it into  $p = 1 - q$ . So  $p = 1 - 0.39$  or 0.61. Remember that the homozygous dominant genotype is AA or  $p^2$ . You know p so square it to find  $p^2$ :  $0.61 \times 0.61 = 0.3721$ . To round out the calculations you could compute Aa, the heterozygotes. Aa is the same as  $2pq$  or  $2(0.39)(0.61) = 0.4758$ . If asked for the homozygous recessive ( $aa - q^2$ ) you would square  $0.39 = 0.1521$ . The last two were not required in the problem, but I wanted you to see how to get them. Now for some work of your own.

**Type III - Sample problems:**

1. In a population in Hardy-Weinberg equilibrium, there are two alleles, A and a, for a particular gene. If the frequency of the a allele is 0.01, what is the frequency of genotype Aa?
2. In a population in Hardy-Weinberg equilibrium, there are two alleles, B and b, for a particular gene. If the frequency of the B allele is 0.6, what is the frequency of genotype Bb? (hint you know p this time!)
3. In a population in Hardy-Weinberg equilibrium, there are two alleles, R and r, for a particular gene. If the frequency of the r allele is 0.3, what is the frequency of genotype RR?

